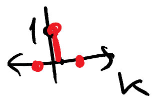
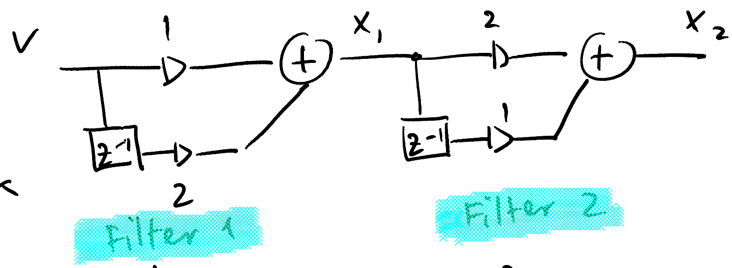
Solusi Quis I PSDL&A

(1) r_{vv}





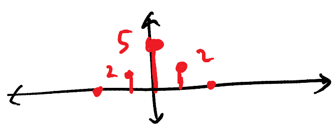
$r_{x_1x_1}(0)$:

1 2
1 2 ×
1 + 4 = 5

$r_{x_1x_1}(1)$:

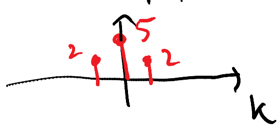
1 2
1 2 ×
2 = 2

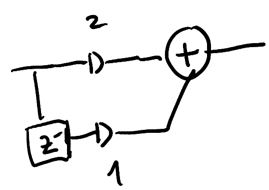
$r_{x_1x_1}$



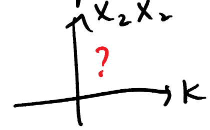
⇒ $r_{x_1x_1}$ menjadi input filter 2:

$r_{x_1x_1}(k)$

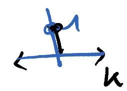




$r_{x_2x_2}$



response filter 2 terhadap



2 1
2 1 ×
4 + 1 = 5

2 1
2 1 ×
2 = 2

Sehingga fungsi auto korelasi

$r_{x_2x_2}$: $r_{x_2x_2}(0)$:

2 5 2
2 5 2
4 + 25 + 4 = 33

$r_{x_2x_2}(1)$:

2 5 2
2 5 2 *
10 + 10 = 20

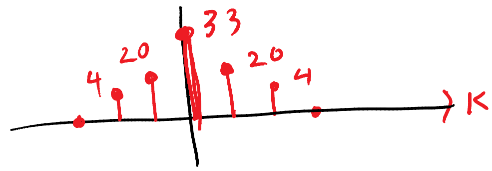
$r_{x_2x_2}(2)$:

2 5 2
2 5 2 ×
4 = 4

$r_{x_2x_2}(k)$

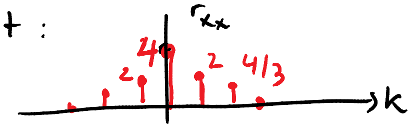
Sehingga $r_{x_2x_2}$:

Sehingga $r_{x_2x_2}$:



2). Diberikan Proses AR(2) dengan input gaussian.

output :



a) Pers Yule-Walker dari AR(2)

$$\begin{pmatrix} r_{xx}(0) & r_{xx}(-1) & r_{xx}(-2) \\ r_{xx}(1) & r_{xx}(0) & r_{xx}(1) \\ r_{xx}(2) & r_{xx}(1) & r_{xx}(0) \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_0^2 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_0^2 \\ 0 \\ 0 \end{pmatrix}$$

b) Modified Yule-Walker :

$$\begin{pmatrix} r_{xx}(0) & r_{xx}(1) \\ r_{xx}(1) & r_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -r_{xx}(1) \\ -r_{xx}(2) \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4/3 \end{pmatrix}$$

c) Iterasi Levinson-Durbin,

Mulai dari AR(1):

$$\begin{pmatrix} r_{xx}(0) & r_{xx}(-1) \\ r_{xx}(1) & r_{xx}(0) \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \end{pmatrix} = \begin{pmatrix} b_0^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b_0^2 \end{pmatrix}$$

$r_{xx}(1) \quad r_{xx}(0)$

$$\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ a_1 \end{pmatrix} = \begin{pmatrix} b_0^2 \\ 0 \end{pmatrix}$$

gunakan baris 2:

$$2 + 4a_1 = 0 \Rightarrow \underline{a_1 = -\tfrac{1}{2}}$$

gunakan baris 1:

$$4 + 2a_1 = b_0^2 \Rightarrow b_0^2 = 4 + 2\left(-\tfrac{1}{2}\right) = \underline{3}$$

Pers Yule-Walker AR(1):

$$\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Ekspasi ke AR(2):

Hitung:

$$A = \begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ \frac{1}{3} \end{pmatrix}$$

dan:

$$B = \begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ -1/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 0 \\ 3 \end{pmatrix}$$

sehingga:

$$A - \frac{1}{9}B = \begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 - \frac{1}{9}\cdot 0 \\ -\frac{1}{2} - \frac{1}{9}\cdot\left(-\frac{1}{2}\right) \\ 0 - \frac{1}{9}\cdot 3 \end{pmatrix} = \begin{pmatrix} 3 - \frac{1}{27} \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -4/9 \\ -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 80/27 \\ 0 \\ 0 \end{pmatrix}$$

bandingkan dengan pers Yule Walker:

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$$\begin{pmatrix} 4 & 2 & 4/3 \\ 2 & 4 & 2 \\ 4/3 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_0^2 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \quad a_1 = -4/9 \qquad b_0^2 = \frac{80}{27}$

$a_2 = -\frac{1}{3} \qquad b_0 = \sqrt{\frac{80}{27}} = \frac{4\sqrt{5}}{3\sqrt{3}}$