Tuesday, October 08, 2019
9:18 AM

# Contoh Latihan ARMA

Suatu filter ARMA (1,1) diberi input Gaussian process dengan mean = 0 dan variansi = 1.
Keluaran filter memiliki $r_{xx}(0) = \frac{4}{3}$ ; $r_{xx}(1) = \frac{1}{3}$ ; $r_{xx}(2) = -\frac{1}{6}$

a) Gambarkan struktur filter ARMA tsb!

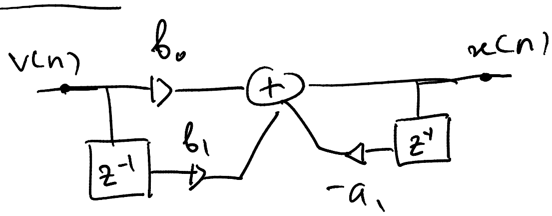
b) Tuliskan pers. Yule Walker yang mewakili proses ARMA tsb !

c) Cari nilai $a_1$ , $c_0$ dan $c_1$

d) Cari nilai $b_0$ dan $b_1$.

Jawab :

a)




ARMA (1,1)

b) Persamaan Yule-Walker :

$$\begin{bmatrix} r_{xx}(0) & r_{xx}(-1) \\ r_{xx}(1) & r_{xx}(0) \\ r_{xx}(2) & r_{xx}(1) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4/3 & \frac{1}{3} \\ 1/3 & 4/3 \\ -\frac{1}{6} & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ 0 \end{bmatrix}$$

c) mencari $a_1$ : ambil baris 3 pers Yule Walker

$$-\frac{1}{6} + \frac{1}{3} a_1 = 0 \Rightarrow a_1 = \frac{1}{2}$$

mencari $c_0$ $\rightarrow$ ambil baris 1 pers YW

$$\frac{4}{3} + \frac{1}{3} a_1 = c_0$$

$$\frac{4}{3} + \frac{1}{3}\left(\frac{1}{2}\right) = c_0 \Rightarrow c_0 = \frac{9}{6} = \frac{3}{2}$$

mencari $c_1$ $\rightarrow$ ambil baris 2 pers YW

$$\frac{1}{3} + \frac{4}{3} a_1 = c_1$$

$$\frac{1}{3} + \frac{4}{3} \cdot \left(\frac{1}{2}\right) = c_1 \Rightarrow c_1 = 1$$

d) Mencari $b_0$ & $b_1$ :

$$A(z) = 1 + a_1 z^{-1} = 1 - \left(\frac{1}{2}\right) z^{-1} = 1 + \frac{1}{2} z^{-1}$$

$$A\left(\frac{1}{z}\right) = 1 + \frac{1}{2} z$$

$$C(z) = c_0 + c_1 z^{-1} = \frac{3}{2} + z^{-1}$$

$$C(z) \cdot \left(A\left(\frac{1}{z}\right)\right) = \left(\frac{3}{2} + z^{-1}\right)\left(1 + \frac{1}{2} z\right)$$

$$= \frac{3}{2} + \frac{3}{4} z + z^{-1} + \frac{1}{2}$$

$$\frac{8}{6} - 3$$

$$= \underbrace{\frac{3}{4} z}_{\text{non Kausal}} + \underbrace{2 + z^{-1}}_{\text{Kausal}}$$

buang bagian Non Kausal, dan paksa Simetri :

$$B(z) \cdot B\left(\frac{1}{z}\right) = z + 2 + z^{-1}$$

$$= (A + Bz^{-1})(A + Bz \quad )$$

$$= (1 + z^{-1})(1 + z \quad )$$

$$B(z) = 1 + z^{-1} \rightarrow b_0 = 1$$

$$\underline{\underline{b_1 = 1}}$$