Introduction to Matching Pursuit (MP)

2nd edition

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Introduction

This tutorial is a second edition from the first one in May 2014.

1. The original title was OMP, but it was wrong. The correct one is MP.
2. MP is popularized by Mallat and Zhang, in their paper : *Mallat and Zhang, 1993, Matching Pursuits With Time-Frequency Dictionaries, IEEE Transactions on Signal Processing, Number 12, Volume 41.*
3. if you find this tutorial useful, please cite: Usman, Koredianto, 2017, Introduction to Matching Pursuit, Online: *http://korediantousman.staff.telkomuniversity.ac.id/tutorial. Access on : your access time.*

1. Problem Statement

Consider the following very simple example: Given the following sparse signals

$$x = \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

The following is the measurement matrix A (2 x 3):

$$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$$

Therefore $y = A \cdot x$ gives:

$$y = \begin{pmatrix} -1.65 \\ -0.25 \end{pmatrix}$$

Now, Given that : $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$ and $A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$

How to find original **x**?

2. Orthogonal Matching Pursuit

BASIS Previous example: Given : $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$ and

$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$

Columns in matrix A are called BASIS (CHEN and DONOHO : **ATOMS**). In the example, we have the following *atoms*:

$$b_1 = \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

Interpretation of equation Ax = y
Since A = $[b_1\ b_2\ b_3]$; and if we let : x = $[a\ b\ c]$, then
$A \cdot x = a \cdot b_1 + b \cdot b_2 + c \cdot b_3$ $A \cdot x$ is the linear combination of b_1, b_2, b_3.

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 0 \\ 1 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

From the equation above, it is clear that atom b_1 contribute the biggest influence in y, next is b_2, dan last is b_3. ORTHOGONAL MATCHING PURSUIT works reversely: we start finding which of b_1, b_2, b_3 that will influence the STRONGEST to y. Then the SECOND STRONGEST from the residual, and so on.

2. Orthogonal Matching Pursuit

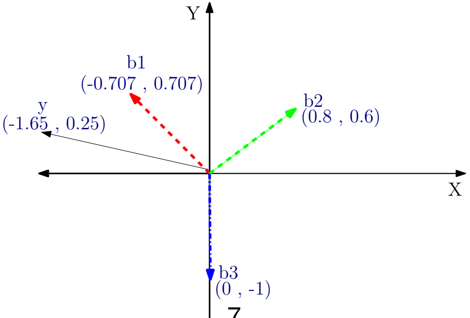
STRONGEST influence is measured using DOT PRODUCT / INNER PRODUCT OMP Algorithm:

1. find atom with has the biggest inner product with y

$$p_i = max_j < b_j, y >$$

2. calculate the residue $r_i = p_i - p_i \cdot < p_i, y >$
3. find atom with has the biggest inner product with r_i
4. repeat step 2 and 3 until residue achieve a certain threshold

Geometrically:



Here the dot product of y to any of b1, b2, b3:

$$< y, b_1 >= -1.34$$

$$< y, b_2 >= 1.17$$

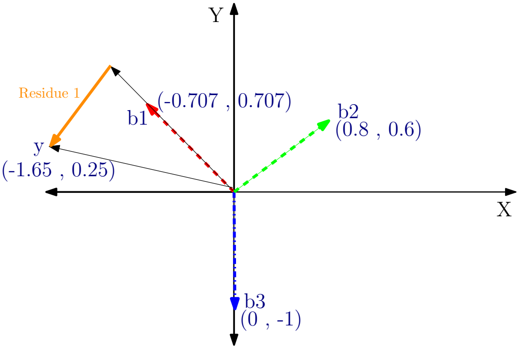
and

$$< y, b_3 >= 0.25$$

Taking the absolute value, we see b_1 gives the biggest inner product. Then, b_1 is chosen as the atom in first step, DOT PRODUCT -1.34. We next count the residue:

$$r_1 = y - b_1 \cdot < y, b_1 > = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix} - (-1.34) \cdot \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

2. Orthogonal Matching Pursuit



Next we count the DOT PRODUCT of this residue to b_2 and b_3 (no need to count with b_1, since this residue must perpendicular to b_1).

$$< r1, b1 >= 1$$

$$< r1, b3 >= -0.7$$

Taking the absolute value, we get b_2 as the next strongest influence.

2. Orthogonal Matching Pursuit

Next we count again the residue:

$$r2 = r1 - < r1, b2 > b2 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix} - (1) \cdot \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$$

$$= \begin{pmatrix} -0.099 \\ 0.099 \end{pmatrix}$$

From residue r2, we finally count the final DOT PRODUCT, between r2 with the last b_3 :

$$< r2, b3 >= -0.099$$