Introduction to Orthogonal Matching Pursuit

Koredianto Usman
Telkom University
Faculty of Electrical Engineering
Indonesia

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This tutorial is a continuation of our previous tutorial on Matching Pursuit (MP).

1 Introduction

Consider the following situation. Given

$$\mathbf{x} = \begin{bmatrix} -1.2 \\ 1 \\ 0 \end{bmatrix}$$

and

$$\mathbf{A} = \begin{bmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{bmatrix}$$

Calculate $\mathbf{y} = \mathbf{A} \cdot x$!

Well this is easy. Simply

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x} = \begin{bmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1.2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix}.$$

Now the difficult part. Given

$$\mathbf{y} = \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix}.$$

and

$$\mathbf{A} = \begin{bmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{bmatrix},$$

find the original (or as close as possible to) $\mathbf{x}$!

In Compressive Sensing terminology, finding $\mathbf{y}$ from $\mathbf{x}$ and $\mathbf{A}$ is called **compression**. While, on the other hand, finding original $\mathbf{x}$ from $\mathbf{y}$ and $\mathbf{A}$ is called reconstruction (problem). Yes, it is indeed a problem, since reconstruction is not an easy task.

As for further terminology, $\mathbf{x}$ is called **original signal** or **original vector**, $\mathbf{A}$ is called the **ompression matrix** or **sensing matrix**, and $\mathbf{y}$ is called **compressed signal** or **compressed vector**.

2 Basic Concept

As already discussed in MP tutorial, we need to view the sensing matrix $\mathbf{A}$ as a collection of column vectors. That is:

$$\mathbf{A} = \begin{bmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{bmatrix} = \begin{bmatrix} \mathbf{b_1} & \mathbf{b_2} & \mathbf{b_3} \end{bmatrix},$$

where

$$\mathbf{b_1} = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} \quad ; \quad \mathbf{b_2} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \quad ; \quad \mathbf{b_3} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

These columns are called the basis or atoms.

Now, if we let

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then

$$\mathbf{A} \cdot \mathbf{x} = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} \cdot x_1 + \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \cdot x_2 + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot x_3 = \mathbf{y}.$$

The last equation shows us that $\mathbf{y}$ is nothing but a linear combination of atoms using coefficients as described in $\mathbf{x}$. We know that actually $x_1 = -1.2$, $x_2 = 1$ and $x_3 = 0$. In other word, looking to the last equation atom $\mathbf{b_1}$ contributes -1.2 to produce $\mathbf{y}$, atom $\mathbf{b_2}$ contributes 1 to produce $\mathbf{y}$, and finally, atom $\mathbf{b_3}$ contributes 0 to $\mathbf{y}$.

OMP works similar to MP, which is to find which atom contributes the most to $\mathbf{y}$, and then which one is the next, and which one is the next, and so on. This process, as we understand by now, need N iteration, where N is the number of atoms in $\mathbf{A}$. In our example, N is 3.

3 Calculating Contribution

There are three atoms, which are:

$$\mathbf{b_1} = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} \quad ; \quad \mathbf{b_2} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \quad ; \quad \mathbf{b_3} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

and $\mathbf{y}$

$$\mathbf{y} = \begin{bmatrix} -1.65 \\ -0.25 \end{bmatrix}.$$

Therefore the contribution for each atom in $\mathbf{y}$ is

$$\langle \mathbf{b_1}, \mathbf{y} \rangle = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} \cdot \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} = -0.707 \cdot -1.65 + 0.707 \cdot -0.25 = -1.34$$

and

$$\langle \mathbf{b_2}, \mathbf{y} \rangle = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \cdot \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} = 1.17$$

and finally

$$\langle \mathbf{b_3}, \mathbf{y} \rangle = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} = 0.25$$

Obviously, atom $\mathbf{b_1}$ gives the highest contribution, with the value of **-1.34** (neglect the negative value or just use the absolute value)

To in this first round iteration, we select $\mathbf{b_1}$ as the selected basis, and the coefficient of that basis is **-1.34**.

Of course, the dot product of each basis can be calculate in a single step, $\mathbf{w}$, which is:

$$\mathbf{w} = \mathbf{A}^T \cdot \mathbf{y} = \begin{bmatrix} -0.707 & 0.707 \\ 0.8 & 0.6 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} = \begin{bmatrix} -1.34 \\ 1.17 \\ 0.25 \end{bmatrix}$$

Figure 1 shows the illustration of basis and $\mathbf{y}$.

4 Calculating Residue

Now the first basis been selected, which is $\mathbf{b_1} = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}$, and the coefficient $\lambda_1 = -1.34$ as the contribution coefficient. If we take $\lambda_1 \cdot \mathbf{b_1}$

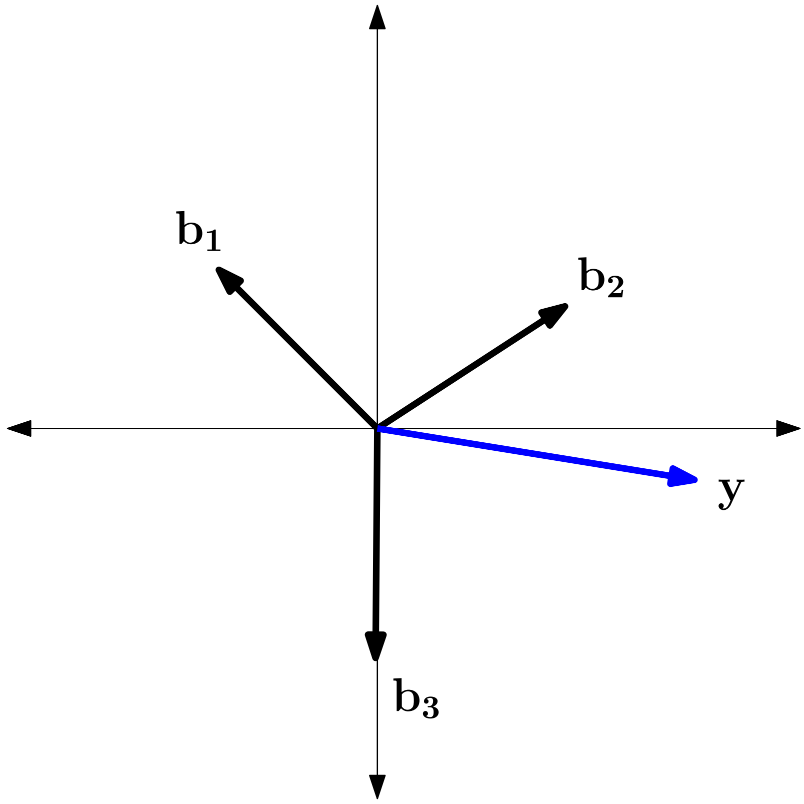

Figure 1: Illustration of basis and $\mathbf{y}$

from $\mathbf{y}$, the residue would be:

$$\mathbf{r} = \mathbf{y} - \lambda_1 \cdot \mathbf{b_1} = \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} - (-1.34) \cdot \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}$$

What does this residue mean to us? Well, let see again the geometry as shown in Fig.2.

As we can see, the residue is perpendicular to the contribution of the first selected basis.

5 Repeat the iteration

After the first iteration, we get the selected basis $\mathbf{b_1}$. We collect this selected basis into a new $\mathbf{A_{new}}$. That is

$$\mathbf{A_{new}} = [\mathbf{b_1}] = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}.$$

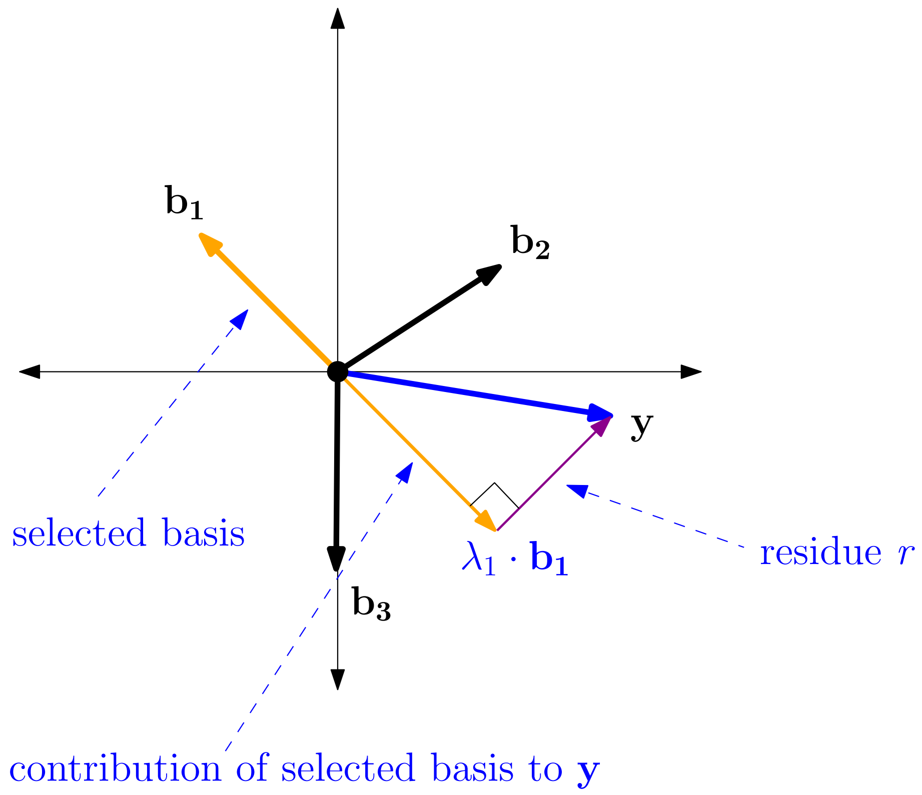

Figure 2: Interpretation of residue

The contribution coefficient as we rewrite it as $\mathbf{x}_{rec}$ is

$$\mathbf{x_{rec}} = \begin{bmatrix} -1.34 \\ 0 \\ 0 \end{bmatrix}.$$

The value -1.34 is at the first element because it is a contribution from the first basis $\mathbf{b_1}$.

The residue is

$$\mathbf{r} = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}.$$

Now we have to select from the rest of basis $\mathbf{b_2}$ and $\mathbf{b_3}$, which one has the highest contribution to $\mathbf{r}$. In single step

$$\mathbf{w} = \begin{bmatrix} \mathbf{b_2} & \mathbf{b_3} \end{bmatrix}^T \cdot r = \begin{bmatrix} 0.8 & 0.6 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 0.98 \\ -0.7 \end{bmatrix}$$

So here $\mathbf{b_2}$ contribute better which is 0.98.

Now we collect the selected basis $\mathbf{b_1}$ and $\mathbf{b_2}$ into one matrix $\mathbf{A_{new}}$, so that

$$\mathbf{A_{new}} = \begin{bmatrix}\mathbf{b_1} & \mathbf{b_2}\end{bmatrix} = \begin{bmatrix}-0.707 & 0.8\\ 0.707 & 0.6\end{bmatrix}.$$

Now, this step is different with MP. Here, we have to calculate the contribution of $\mathbf{A_{new}}$ to $\mathbf{y}$ (and not the contribution of $\mathbf{b_2}$ to $\mathbf{r}$!).

To solve this contribution problem, we have to solve the Least Square Problem which can be easily formulated as follow:

Which λ_1 dan λ_2 that will fulfilled

$$\lambda_1 \cdot \begin{bmatrix}-0.707\\ 0.707\end{bmatrix} + \lambda_2 \cdot \begin{bmatrix}0.8\\ 0.6\end{bmatrix} \textit{ as close as possible to } \mathbf{y} = \begin{bmatrix}1.65\\ -0.25\end{bmatrix}$$

in mathematical term this formulation can be written as :

$$min \left\| \mathbf{A_{new}} \cdot \boldsymbol{\lambda} - \mathbf{y} \right\|_2$$

In our case

$$min \left\| \begin{bmatrix}-0.707 & 0.8\\ 0.707 & 0.6\end{bmatrix} \cdot \begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} - \begin{bmatrix}1.65\\ -0.25\end{bmatrix} \right\|_2$$

Reader may familiar that $min \left\| \mathbf{A_{new}} \cdot \boldsymbol{\lambda} - \mathbf{y} \right\|_2$ is solve using $\boldsymbol{\lambda}$

$$\boldsymbol{\lambda} = \mathbf{A_{new}}^{+} \cdot \mathbf{y}$$

Where $\mathbf{A_{new}}^{+}$ is pseudo inverse of $\mathbf{A_{new}}$, which is

$$\mathbf{A_{new}}^{+} = (\mathbf{A_{new}}^{T} \cdot \mathbf{A_{new}})^{-1} \cdot \mathbf{A_{new}}^{T}.$$

Again, in our case,

$$\mathbf{A_{new}}^{+} = \begin{bmatrix}-0.707 & 0.8\\ 0.707 & 0.6\end{bmatrix}^{+} = \begin{bmatrix}-0.6062 & 0.8082\\ 0.7143 & 0.7143\end{bmatrix}$$

Calculating pseudo inverse is easy in Matlab, just use `pinv()` command.

After calculating $\mathbf{A_{new}}^{+}$, we get

$$\boldsymbol{\lambda} = \mathbf{A_{new}}^{+} \cdot \mathbf{y} = \begin{bmatrix}-0.6062 & 0.8082\\ 0.7143 & 0.7143\end{bmatrix} \cdot \begin{bmatrix}1.65\\ -0.25\end{bmatrix} = \begin{bmatrix}-1.2\\ 1\end{bmatrix}$$

As we get $\boldsymbol{\lambda}$, we update coefficient $\mathbf{x_{rec}}$ to become

$$\mathbf{x_{rec}} = \begin{bmatrix}-1.2\\ 1\\ 0\end{bmatrix}$$

Here we put $\boldsymbol{\lambda}$ in the first and second position for $\mathbf{x_{rec}}$ because it corresponds to the first and second basis that have been selected.

After we get $\mathbf{A_{new}}$ and $\boldsymbol{\lambda}$, now we can update new the residue

$$\mathbf{r} = \mathbf{y} - \mathbf{A_{new}} \cdot \boldsymbol{\lambda} = \begin{bmatrix} 1.65 \\ -0.25 \end{bmatrix} - \begin{bmatrix} -0.707 & 0.8 \\ 0.707 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} -1.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Well, our residue is already $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. So we can stop here or we can continue to the next step. (If we stop here, we can save some computation work)

6 Final iteration

This step is not necessary as the residue is already vanished. (Many OMP implemetation software requires parameter which is signal sparsity K. This is the clue for the software that it will iterate K times. After that it will stop, no matter how much the residue left).

Our reconstructed signal is therefore :

$$\mathbf{x_{rec}} = \begin{bmatrix} -1.2 \\ 1 \\ 0 \end{bmatrix}$$

Which is the same to the original signal.

7 Other example

Now we proceed with other example. Let

$$\mathbf{x} = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

and

$$\mathbf{A} = \begin{bmatrix} -0.8 & 0.3 & 1 & 0.4 \\ -0.2 & 0.4 & -0.3 & -0.4 \\ 0.2 & 1 & -0.1 & 0.8 \end{bmatrix}.$$

Having $\mathbf{x}$ and $\mathbf{A}$, $\mathbf{y}$ is calculated as

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x} = \begin{bmatrix} -0.8 & 0.3 & 1 & 0.4 \\ -0.2 & 0.4 & -0.3 & -0.4 \\ 0.2 & 1 & -0.1 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2.7 \\ 0.1 \\ 4.5 \end{bmatrix}$$

Now, given $\mathbf{A}$ and $\mathbf{y}$, please find $\mathbf{x}$ using OMP!

Ans

1. We have 4 basis :
$$\mathbf{b_1} = \begin{bmatrix} -0.8 \\ -0.2 \\ 0.2 \end{bmatrix} ; \mathbf{b_2} = \begin{bmatrix} 0.3 \\ 0.4 \\ 1 \end{bmatrix} ; \mathbf{b_3} = \begin{bmatrix} 1 \\ -0.3 \\ -0.1 \end{bmatrix} ; \mathbf{b_4} = \begin{bmatrix} 0.4 \\ -0.4 \\ 0.8 \end{bmatrix}$$
2. Since the length of basis is not one, we need to normalize them all.
$$\hat{\mathbf{b_1}} = \mathbf{b_1} / \|\mathbf{b_1}\| = \begin{bmatrix} -0.8 \\ -0.2 \\ 0.2 \end{bmatrix} / \sqrt{(-0.8)^2 + (-0.4)^2 + 0.2^2} = \begin{bmatrix} -0.9428 \\ -0.2357 \\ 0.2357 \end{bmatrix}$$
$$\hat{\mathbf{b_2}} = \mathbf{b_2} / \|\mathbf{b_2}\| = \begin{bmatrix} 0.3 \\ 0.4 \\ 1 \end{bmatrix} / \sqrt{0.3^2 + 0.4^2 + 1^2} = \begin{bmatrix} 0.268 \\ 0.3578 \\ 0.894 \end{bmatrix}$$
$$\hat{\mathbf{b_3}} = \mathbf{b_3} / \|\mathbf{b_3}\| = \begin{bmatrix} 0.9535 \\ -0.286 \\ -0.0953 \end{bmatrix}$$
$$\hat{\mathbf{b_4}} = \mathbf{b_4} / \|\mathbf{b_4}\| = \begin{bmatrix} 0.4082 \\ -0.4082 \\ -0.8165 \end{bmatrix}$$
3. contribution of these normalize basis can be calculated as:
$$\mathbf{w} = \hat{\mathbf{A}}^T \cdot \mathbf{y} = \begin{bmatrix} -0.9428 & 0.268 & 0.9535 & 0.4082 \\ -0.2357 & 0.3578 & -0.286 & -0.4082 \\ 0.2357 & 0.894 & -0.0953 & -0.8165 \end{bmatrix} \cdot \begin{bmatrix} 2.7 \\ 0.1 \\ 4.5 \end{bmatrix} = \begin{bmatrix} -1.5085 \\ 4.7852 \\ 2.1167 \\ 4.7357 \end{bmatrix}$$

4. Second normalized basis $\hat{\mathbf{b_2}}$ gives the highest contribution. Therefore we take second basis and put it to $\mathbf{A_{new}}$
$\mathbf{A_{new}} = \hat{\mathbf{b_2}} = \begin{bmatrix} 0.3 \\ 0.4 \\ 1 \end{bmatrix}$.

5. Calculte $\mathbf{x_{rec}}$ as the solution of Least Square Problem:

$$\mathbf{L_p} = \mathbf{A_{new}}^+ \cdot \mathbf{y} = \begin{bmatrix} 4.28 \end{bmatrix}$$

Since this $\mathbf{Lp}$ is the coefficient for basis 2, then we write

$$\mathbf{x_{rec}} = \begin{bmatrix} 0 \\ 4.28 \\ 0 \\ 0 \end{bmatrix}$$

6. Next we calculate residue

$$\mathbf{r} = \mathbf{y} - \mathbf{A_{new}} \cdot \mathbf{L_p} = \begin{bmatrix} 2.7 \\ 0.1 \\ 4.5 \end{bmatrix} - \begin{bmatrix} 0.3 \\ 0.4 \\ 1 \end{bmatrix} \cdot 4.28 = \begin{bmatrix} 1.416 \\ -1.612 \\ 0.22 \end{bmatrix}$$

7. Now we repeat step 2. We calculate which basis $\hat{\mathbf{b_1}}$, $\hat{\mathbf{b_3}}$ and $\hat{\mathbf{b_4}}$ that gives highest contribution to $\mathbf{r}$.

$$\mathbf{w} = \hat{\mathbf{A}}^T \cdot \mathbf{y} = \begin{bmatrix} -0.9428 & -0.2357 & 0.2357 \\ 0.9535 & -0.286 & -0.0953 \\ 0.4082 & -0.4082 & -0.8165 \end{bmatrix} \cdot \begin{bmatrix} 1.416 \\ -1.612 \\ 0.22 \end{bmatrix} = \begin{bmatrix} -0.9032 \\ 1.7902 \\ 1.4158 \end{bmatrix}$$

So here, at the second position which corresponding to $\mathbf{b_3}$ gives the highest contribution 1.7902.

8. Add this the selected $\mathbf{b_3}$ (not $\hat{\mathbf{b_3}}$) in to $\mathbf{A_{new}}$

$$\mathbf{A_{new}} = \begin{bmatrix} \hat{\mathbf{b_2}} & \hat{\mathbf{b_3}} \end{bmatrix} = \begin{bmatrix} 0.3 & 1 \\ 0.4 & -0.3 \\ 1 & -0.1 \end{bmatrix}.$$

9. Solve the Least Square Problem for $\mathbf{Lp}$
$\mathbf{L_p} = \mathbf{A_{new}}^+ \cdot \mathbf{y} = \begin{bmatrix} 4.1702 \\ 1.7149 \end{bmatrix}$

We know that this **Lp** is for $\mathbf{b_2}$ and $\mathbf{b_3}$, therefore we write

$$\mathbf{x_{rec}} = \begin{bmatrix} 0 \\ 4.172 \\ 1.7149 \\ 0 \end{bmatrix}$$

10. Next we calculate residue

$$\mathbf{r} = \mathbf{y} - \mathbf{A_{new}} \cdot \mathbf{L_p} = \begin{bmatrix} 2.7 \\ 0.1 \\ 4.5 \end{bmatrix} - \begin{bmatrix} 0.3 & 1 \\ 0.4 & -0.3 \\ 1 & -0.1 \end{bmatrix} \cdot \begin{bmatrix} 4.172 \\ 1.7149 \end{bmatrix} = \begin{bmatrix} -0.266 \\ -1.0536 \\ 0.5012 \end{bmatrix}$$

11. Now we repeat step 2 as for the final iteration.

12. calculate highest contribution from either $\mathbf{b_1}$ or $\mathbf{b_4}$

$$\mathbf{w} = \begin{bmatrix} \hat{\mathbf{b}}_\mathbf{1} & \hat{\mathbf{b}}_\mathbf{4} \end{bmatrix} \cdot \mathbf{r} = \begin{bmatrix} -0.9428 & 0.4082 \\ -0.2357 & -0.4082 \\ 0.2357 & -0.8165 \end{bmatrix} \cdot \begin{bmatrix} -0.266 \\ -1.0536 \\ 0.5012 \end{bmatrix} = \begin{bmatrix} 0.6172 \\ 0.7308 \end{bmatrix}$$

So here, at the second position which corresponding to $\mathbf{b_4}$ gives the highest contribution 0.7308.

13. And then collect this basis to already available $\mathbf{A_{new}}$

$$\mathbf{A_{new}} = \begin{bmatrix} 0.3 & 1 & 0.4 \\ 0.4 & -0.3 & -0.4 \\ 1 & -0.1 & 0.8 \end{bmatrix}.$$

14. Solve the Least Square Problem for **Lp**

$$\mathbf{L_p} = \mathbf{A_{new}}^+ \cdot \mathbf{y} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

We know that this **Lp** is for $\mathbf{b_2}$, $\mathbf{b_3}$, and $\mathbf{b_4}$, therefore we write

$$\mathbf{x_{rec}} = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

15. Next we calculate residue

$$\mathbf{r} = \mathbf{y} - \mathbf{A_{new}} \cdot \mathbf{L_p} = \begin{bmatrix} 2.7 \\ 0.1 \\ 4.5 \end{bmatrix} - \begin{bmatrix} 0.3 & 1 & 0.4 \\ 0.4 & -0.3 & -0.4 \\ 1 & -0.1 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

16. Our iteration can be stopped here, as the residue is already zero. The reconstruction $\mathbf{x}$ in this case is

$$\mathbf{x_{rec}} = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

which is similar to the original $\mathbf{x}$.

8 Final notes

As we see in the ilustration process above, we now may notes the following facts.

- The highest contribution as we calculated from OMP is derived from normalized basis. Not from the original unnormalized basis.
- If all basis in measurement or reconstruction matrix $\mathbf{A}$ has already been normalized, the the calculation can be proceeded without normalization step.
- The contribution by each basis is done by dot product of the **residue** and the **normalize basis**.
- In the case of MP, the reconstruction coefficient x_{rec} is calculated from dot product of basis and residue, while on OMP, the coefficient x_{rec} is calculated from Least Square solution of $\mathbf{A_{new}}$ to $\mathbf{y}$. The solution of this Least Square Problem is actually $\mathbf{L_p}$. And $\mathbf{L_p}$ is directly indicates $\mathbf{x_{rec}}$ with a proper positio. $\mathbf{A_{new}}$ itself is collected from already selected unnormalized basis. This process needs time. Therefore OMP is slower dan MP.
- Residue $\mathbf{r}$ is calculated from original $\mathbf{y}$ and $\mathbf{A_{new}}$ and $\mathbf{L_p}$.
- The Iteration continues as many as the number or row in $\mathbf{A}$ (i.e. M). Or, if the sparsity of the signal k is known, then the iteration repeats k times. If $k <$ M, then knowing k will help our computation faster. However if k is unknown, the iteration should be up to M.

9 Mathematical formal algorithm for OMP

Now, after we understand previous process, we can state the following steps as OMP algorithm.

OMP algorithm

1.

10 Weakness of OMP

OMP are fast. Very fast, as compared to the convex optimization. This is an advantage of greedy algorithm. But OMP also suffers from highly coherency matrix **A**. What is coherency in a matrix? Coherency in matrix A simply indicates similarity between two columns in matrix **A**.

Look at the following two matrix:
$\mathbf{A_1} = \begin{bmatrix} 0.6 & 0.8 & 1 \\ 0.8 & 0.6 & 0 \end{bmatrix}$ and $\mathbf{A_2} = \begin{bmatrix} 0.6 & 0.61 & 1 \\ 0.8 & 0.79 & 0 \end{bmatrix}$
Matrix $\mathbf{A_2}$ has a very high coherency, because second column is very similar to the first column. Matrix $\mathbf{A_1}$ has lesser coherency, because second column doesnot very similar to the first and third column. The coherency value which is denoted by μ is defined as

$$\mu = \max_{i,j;i\neq j} |\langle A(:,i) \cdot (A :,j)\rangle| .$$

The value of μ is between 0 and 1. If μ is very high, then, usually OMP gives a wrong reconstruction result.

Try the following example:

$$\mathbf{x} = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$$

$$\mathbf{A} = \mathbf{A_2} = \begin{bmatrix} 0.6 & 0.61 & 1 \\ 0.8 & 0.79 & 0 \end{bmatrix}$$

Find **y**.

Then given **A** and **y**, using OMP, find **x** back! Can you get the original **x**?

11 Cite this document

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